Mat 2377

June 21, 2016

Solution 4 on (34)

4.8 ($2points$)

a) $\bar{x} = 11.69$ mg

b) $s^2 = \frac{n\sum x_i^2 - \left(\sum x_i\right)^2}{n(n-1)} = \frac{8(1168.21) - 93.5^2}{8(7)} = 10.776$

4.20 ($6points$)$n = 64, \mu_X = 3.2, \sigma_X = \sigma/\sqrt{n} = 1.6/8 = 0.2$

a) $z = (2.7 - 3.2)/0.2 = -2.5; P\left(\bar{X} < 2.7\right) = P(Z < -2.5) = 0.0062$

b) $z = (3.5 - 3.2)/0.2 = 1.5; P\left(\bar{X} > 3.5\right) = P(Z > 1.5) = 0.0668$

c) $z_1 = (3.2 - 3.2)/0.2 = 0; z_2 = (3.4 - 3.2)/0.2 = 1;$

$P\left(3.2 < \bar{X} < 3.4\right) = P(0 < Z < 1) = 0.8413 - 0.5000 = 0.3413$

4.24 ($3points$)

(a) $n_1 = n_2 = 36$; $z = 0.2/\sqrt{1/36 + 1/36} = 0.85$

$$P\left(\bar{X}_B - \bar{X}_B \geq 0.2\right) = P(Z \geq 0.85) = 0.1977$$

(b) Since the probability in a) is not negligible, the experiments do not strongly support teh conjecture

4.40 ($3points$)$\bar{x} = 0.475$;$s^2 = 0.0336.t = (0.475 - 0.5)/0.0648 = -0.39$

$P\left(\bar{X} < 0.475\right) = P(T < -0.39) \approx 0.35$

Hence the result is inconclusive

4.52 ($6points$) $\mu = 5000, \sigma = 400, n = 36$

a) By the Central limit theorem, standardizing $Z = \frac{\bar{X} - 5000}{400/\sqrt{36}}$

$$P\left(4800 < \bar{X} < 5200\right) = P(-3 < Z < 3) \approx 0.9974$$

b) $We\mu = \lambda t = 2\,(5) = 10.$want z such that $P\,(-z < Z < z) = 0.99$. Note

$$P\,(Z < z) = 0.995$$

implies $z = 2.575$. Solving $2.575 = \frac{5100-5000}{400/\sqrt{n}}$ implies $n \geq 107$

5.2 $(2points)$ $n = 30, \bar{x} = 780, \sigma = 40$. Also$z_{0.02} = 2.054$. Hence a 96% confidence interval for the populatin mean is

$$780 \pm 2.054\left(40/\sqrt{30}\right)$$

That is $(765, 795)$.

5.6 $(2points)$ $n = [2.05\,(40)\,/10]^2 \approx 35$

5.26 $(3points)$ $n_A = 50 = n_B$; $\bar{x}_A = 78.3, \bar{x}_B = 87.2; \sigma_A = 5.6, \sigma_B = 6.3$. Since $z_{0.025} = 1.96$, a 95% confidence interval for the difference $\mu_A - \mu_B$ is

$$(87.2 - 78.3) \pm 1.96\sqrt{\frac{5.6^2}{50} + \frac{6.3^2}{50}}$$

That is 8.9 ± 2.34 or $(6.56, 11.24)$

5.28$(3points)$ $n_1 = 10, n_2 = 10$; $\bar{x}_1 = 0.399, \bar{x}_2 = 0.565; s_1 = 0.07279, s_2 = 0.18674$. Since $t_{0.025} = 2.101$,with 18 degrees of freedom and the pooled estimate of variance is $s_P = 0.14172$ a 95% confidence interval for the difference $\mu_A - \mu_B$ is

$$(0.565 - 0.399) \pm 2.101\,(0.14172)\,\sqrt{\frac{1}{10} + \frac{1}{10}}$$

That is 0.166 ± 0.133 or $(0.033, 0.299)$

5.44 $(4points)$ $n = 100, \hat{p} = \frac{24}{100} = 0.24, z_{0.005} = 2.575$

a) $0.24 \pm 2.575\sqrt{\frac{0.24(1-0.24)}{100}}$Hence 0.24 ± 0.110 or $(0.130, 0.350)$

b) Error$\leq 2.575\sqrt{\frac{0.24(1-0.24)}{100}} = 0.110$